Spin Rate Variations of Spin-Stabilized Apogee Motors in Thrust Phase

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Nomenclature

C_F = skin-friction coefficient
I_s = roll moment of inertia of solid
$\dot{I}_s$ = rate of change of I_s
k = exit plane radius of gyration for roll
L = nozzle length
$\dot{m}$ = mass flow rate
r = radial distance
r_c = radius of forced vortex core
R = nozzle radius at a given section
T = torque
V_a = axial velocity
V_t = tangential velocity
x = axial coordinate
ω_i = initial spin rate of solid
ω_s = spin rate of solid, $\omega_{\rm solid}$
$\omega_{\rm gas}$ = effective spin rate of exhaust gases

Introduction

REPORTED analytical studies and numerical simulations of the thrust-phase dynamics of the spinning upper stages of launch vehicles are based on one of the two extreme assumptions regarding the jet damping moment along spin axis, namely, the spin rate is constant[1] and the exhaust gases rotate with the same angular velocity as the solid body.[2-4] However, the spin rate is not constant—flight observations show variations in the spin rate initially imparted during the apogee stages. On the other hand, the assumption that $\omega_{\rm gas} = \omega_{\rm solid}$ leads to high final spin rate values at the burnout of typical apogee motors, often showing an increase on the order of 50% or more over the initially imparted spin rate.[4,5] Flight measurements of the spin rate histories, however, indicate only small increases in the spin rate, on the order of 10% or less.[4,6] In Ref. 4, it has been conjectured that neglecting the roll moment of inertia of the chamber gases and its derivatives was responsible for the high spin rate predictions. But, the order-of-magnitude analysis as well as the detailed simulations in Refs. 5 and 7 show that these terms have negligible effect.

The aim of this Note is to show that the observed variations in the spin rate during the thrusting phase of spin-stabilized solid apogee stages can be explained through a model of fluid dynamic viscous interactions between the gaseous products of the motor and the solid surfaces. To the knowledge of the authors, such an approach explaining the spin rate variations has not been reported previously. Even though Breuer and Southerland[8] have analyzed fluid dynamic effects on pitch and yaw jet damping, their study is not applicable to the roll jet damping of spinning motors. Roll jet damping, unlike damping in the pitch and yaw directions, is due to viscous interactions and is not an inviscid phenomena. In spin-stabilized rocket motors, the inviscid flow itself can be very complex and has been the theme of continued efforts, such as the one-dimensional theory of Carpenter and Johannesen[9] and the three-dimensional simulations of Brown and Hoffman.[10] Brown and Hoffman present the torque on the nozzle due to nozzle and missile rotations, but this applied to the pitch and yaw directions and not to roll jet damping.

This paper does not attempt to present an accurate description of the viscous swirling flow and viscous forces on the apogee motor internal surface to explain the spin rate variations. This is a complex fluid dynamics problem and a flight dynamicist, interested in repeated simulations of apogee stage dynamics, requires simpler models if possible. To this end, we propose an approximate method of evaluating viscous torque from nonswirling boundary layers for two extreme inviscid solutions of the forced vortex and free vortex. This is then integrated in time to obtain spin rate histories. The results show good qualitative agreement with reported flight observations.

Methodology

Neglecting thrust misalignment, mass unbalance, and other asymmetric effects, the equation essentially representing the spin rate variations can be written as

$$I_s \frac{d\omega_s}{dt} = \dot{m}k^2\omega_{\rm gas} - \dot{I}_s\omega_s \tag{1}$$

To characterize $\omega_{\rm gas}$, one requires information on the tangential velocity distribution at the nozzle exit plane. Instead, if the visicous torque, along the spin axis T, exerted by the gases on the solid body is known, Eq. (1) can be equivalently written as

$$I_s \frac{d\omega_s}{dt} = T(t) \tag{2}$$

$T(t)$ can be obtained by the knowledge of the swirling boundary layer in the nozzle and evaluating the moments of the skin-friction force about the spin axis.

It is seen that the elemental circumferential component dF of the skin friction is given by

$$dF\,(\text{tangential}) = C_f(x)\frac{\rho}{2}V_{t,\rm rel}\sqrt{V_a^2 + V_{t,\rm rel}^2}\,ds \tag{3}$$

In swirling flows, the axial velocity V_a at a section can be a function of radial distance r. From the solutions of Carpenter and Johannesen,[9] it can be seen that for the spin rate relevant to spin-stabilized apogee motors, V_a can be considered to be uniform at a given axial section.

The relative tangential velocity at the body surface $V_{t,\rm rel}$ is defined by

$$V_{t,\rm rel} = V_t(R) - R\omega_s \tag{4}$$

The inviscid tangential velocity profile $V_t(r)$ can be obtained for a Rankine combined vortex (Fig. 1). Here, we make the observation that the inviscid angular momentum flux imparted to the gases at each instant of time is also equal to the term $\dot{I}\omega_s$ in Eq. (1). This leads to simplified expressions for C_1 and C_2 in Fig. 1, relating them to the moment of inertia variations of the motor as

$$C_1 = \frac{C_2}{r_c^2}, \qquad C_2 = \frac{2\dot{I}_s\omega_s R^2}{\dot{m}(2R^2 - r_c^2)} \tag{5}$$

The spin rate equation now becomes

$$I_s\frac{d\omega_s}{dt} = \dot{m}\int_0^L C_f(x)\,V_{t,\rm rel}\sqrt{1 + \left(\frac{V_{t,\rm rel}}{V_a}\right)^2} \times\sqrt{1 + \left(\frac{dR}{dx}\right)^2}dx \tag{6}$$

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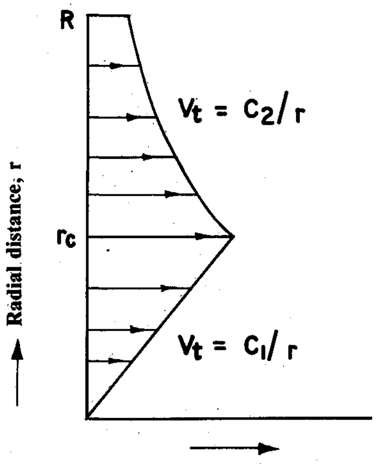

Fig. 1 Tangential velocity profiles in Rankine combined vortex.

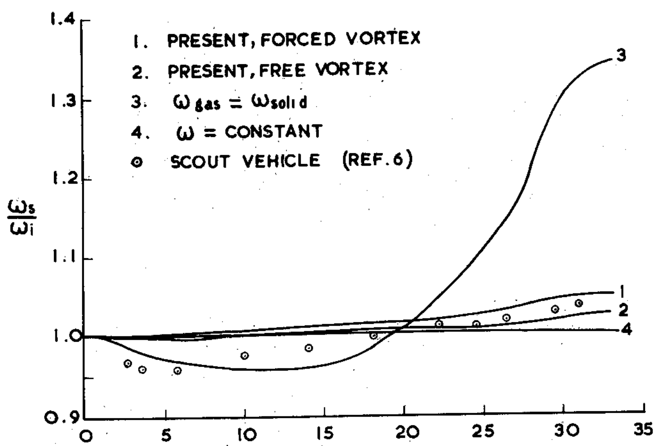

Fig. 2 Comparison of spin rate histories.

Here, the skin-friction contributions from the motor chamber are neglected because they are estimated to be small.[7] $C_f(x)$ has to be evaluated from swirling boundary-layer analysis. But, as a first approximation, we calculate it from a nonswirling boundary-layer code[11] based on a momentum integral approach. This assumption may be justified for moderate swirl from the experimental work of Cannon and Kays,[12] who find that rotation appears to have little measurable effect upon turbulent and laminar flow behavior.

Results

Equation (6) is integrated for 33 s thrusting duration typical of the data for the Scout fourth stage.[4,6] The results are presented in Fig. 2. The present solutions for both the forced and free vortex cases (corresponding to $r_c = R$ and $r_c = 0$, respectively) are given. The computations show that for the free vortex limit, the spin rate is 1.5% more than the initial spin rate. For the forced vortex case, it is about 4.5%. The actual flight measurements show at 33 s an increase of about 3.5%, which is within the other two limits. The flight data show large initial dip compared to present predictions. This may be attributed to simplifying assumptions made for the viscous calculations. In the sharp contrast, the assumption that $\omega_{gas} = \omega_{solid}$ shows a large 35% increase over the initial spin rate and the constant spin rate assumption shows no variation at all. We conclude that, in flight dynamic simulations of spinning apogee motors, the assumption $\omega_{gas} = \omega_{solid}$ should not be made in final spin rate predictions, which are needed for further spacecraft operations. For approximate analysis, the spin rate may be assumed constant, but more accurate predictions should take into account viscous interactions, either in an approximate model as done here, or, if possible, through a complete swirling boundary-layer analysis.

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